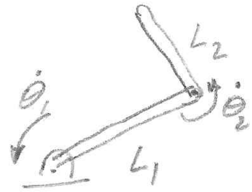
5.1 Repeat Example 5.4 but use ${}^0J(\theta)$.

From Example 5.4 (and 5.3)



$${}^3V_3 = \begin{bmatrix} l_1 s_2 \dot\theta_1 \\ l_1 c_2 \dot\theta_1 + l_2(\dot\theta_1 + \dot\theta_2) \\ 0 \end{bmatrix} \quad \text{so} \quad {}^3J(\theta) = \begin{bmatrix} l_1 s_2 & 0 \\ l_1 c_2 + l_2 & l_2 \end{bmatrix}$$

ALSO

$${}^0V_3 = \begin{bmatrix} -l_1 s_1 \dot\theta_1 - l_2 s_{12}(\dot\theta_1 + \dot\theta_2) \\ l_1 c_1 \dot\theta_1 + l_2 c_{12}(\dot\theta_1 + \dot\theta_2) \\ 0 \end{bmatrix} \text{so} \quad {}^0J(\theta) = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix}$$

To Find singularities

$$\begin{aligned}
\mathrm{DET}({}^0J(\theta)) &= -l_2 c_{12}(l_1 s_1 + l_2 s_{12}) + l_2 s_{12}(l_1 c_1 + l_2 c_{12}) \\
&= -l_1 l_2 s_1 c_{12} - \cancel{l_2^2 s_{12} c_{12}} + l_1 l_2 c_1 s_{12} + \cancel{l_2^2 s_{12} c_{12}} \\
&= l_1 l_2 c_1 s_{12} - l_1 l_2 s_1 c_{12} \\
&= l_1 l_2 (c_1 s_{12} - s_1 c_{12})
\end{aligned}$$

RECALL $s(\theta_1 - \theta_2) = s_1 c_2 - c_1 s_2$

$$\text{SO } s(\overbrace{\theta_1 + \theta_2}^{1 \text{ VARIABLE}} - \theta_1) = s_2$$

$$= s_{12} c_1 - c_1 s_{12}$$

$$= s_2$$

$$\therefore \mathrm{DET}({}^0J(\theta)) = l_1 l_2 s_2$$

singularities are when $\mathrm{DET}(J) = 0$

$$l_1 l_2 s_2 = 0$$

$$s_2 = 0$$

$$\underline{\underline{\theta_2 = 0, 180^\circ}}$$

1 $\quad \mathrm{DET}\ {}^3J(\theta) = l_1 l_2 s_2 \quad \therefore$ SAME singularities for ${}^0J(\theta)$ and ${}^3J(\theta)$

5.11 GIVEN

$$ {}^{A}_{B}T = \begin{bmatrix} 0.866 & -0.5 & 0 & 10 \\ 0.5 & 0.866 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad {}^{A}V = \begin{bmatrix} 0 \\ 2.0 \\ -3.0 \\ 1.414 \\ 1.414 \\ 0 \end{bmatrix} $$

FIND ${}^{B}V$

FROM EQN 5.103

$$ {}^{B}V = \begin{bmatrix} {}^{B}_{A}R & -{}^{B}_{A}R\, {}^{A}P_{X} \\ 0 & {}^{B}_{A}R \end{bmatrix} {}^{A}V $$

$P_X \Rightarrow$ FROM EQN 5.101 WHERE $P = \begin{bmatrix} 10 \\ 0 \\ 5 \end{bmatrix}$

$$ P_X = \begin{bmatrix} 0 & -5 & 0 \\ 5 & 0 & -10 \\ 0 & 10 & 0 \end{bmatrix} $$

$$ {}^{B}_{A}R = \begin{bmatrix} 0.866 & 0.5 & 0 \\ -0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} $$ (NOTICE THE TRANSPOSE)

$$ {}^{B}_{A}R\, P_X = \begin{bmatrix} 0.866 & 0.5 & 0 \\ -0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -5 & 0 \\ 5 & 0 & -10 \\ 0 & 10 & 0 \end{bmatrix} = \begin{bmatrix} 2.5 & -4.3 & -5.0 \\ 4.3 & 2.5 & -8.6 \\ 0 & 10 & 0 \end{bmatrix} $$

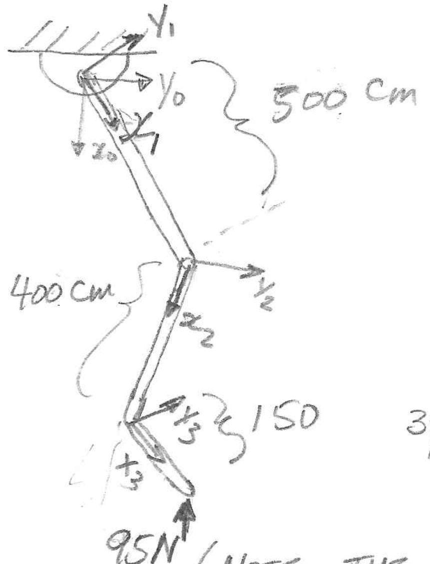
THUS

$$ {}^{B}V = \begin{bmatrix} 0.866 & 0.5 & 0 & -2.5 & 4.3 & 5 \\ -0.5 & 0.866 & 0 & -4.3 & -2.5 & 8.6 \\ 0 & 0 & 1 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0.866 & 0.5 & 0 \\ 0 & 0 & 0 & -0.5 & 0.866 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ -3 \\ 1.414 \\ 1.410 \\ 0 \end{bmatrix} $$

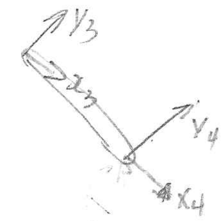
$$ {}^{B}V = \begin{bmatrix} 3.52 \\ -7.8 \\ -17.1 \\ 1.91 \\ 0.51 \\ 0 \end{bmatrix} $$

5.22 USING THE LEG MODEL IN EXCERCISE 3.25, DETERMINE THE JOINT TORQUES, τ, NECESSARY TO COUNTERACT A 95 N FORCE ACTING IN THE VERTICAL DIRECTION AT THE TOE CONTACT POINT IF $\Theta = \begin{bmatrix} 10.5^\circ \\ -44^\circ \\ 3.5^\circ \end{bmatrix}$ (TYPO IN BOOK)

FROM 3.25



$^3P_{TOE} = \begin{Bmatrix} 0 \\ 150 \\ 0 \end{Bmatrix}$



(NOTE, THE ANKLE TO HEEL DISTANCE = 50 IS NOT USED)

TO DEVELOP THE J a 4th FRAME IS ADDED AT THE TOE

TO OBTAIN THE $^4J(\Theta)$ WE MUST OBTAIN VELOCITY IN THE 4th FRAME $^4V_4$

START W/ 0 FRAME

$^0\omega_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad {}^0V_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ THEN WE CAN USE OUR PROPOGATION EQWS FOR VELOCITES + ω's 5.45 + 5.47

BY OBSERVATION $^1\omega_1 = \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 \end{bmatrix} \quad {}^1V_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$^2\omega_2 = {}^2_1R\,{}^1\omega_1 + \dot\theta_2\,{}^2\hat{Z}_2 \qquad {}^1_2R = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 \\ S\theta_2 & C\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 + \dot\theta_2 \end{bmatrix}$$

$$^2V_2 = {}^2_1R\,(\overset{0}{\cancel{^1V_1}} + {}^1\omega_1 \times {}^1P_2) = \begin{bmatrix} C_2 & S_2 & 0 \\ -S_2 & C_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ l_1\dot\theta_1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} l_1 S_2 \dot\theta_1 \\ l_1 C_2 \dot\theta_1 \\ 0 \end{bmatrix}$$

5.22 CONTINUED

$$ {}^3\omega_3 = {}^3_2R\,{}^2\omega_2 + \dot\theta_3\,{}^3\hat{Z}_3 = \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1+\dot\theta_2+\dot\theta_3 \end{bmatrix} $$

$$ {}^3V_3 = {}^3_2R\left({}^2V_2 + {}^2\omega_2 \times {}^2P_3\right) = \begin{bmatrix} C_3 & S_3 & 0 \\ -S_3 & C_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_1 S_2\dot\theta_1 \\ l_1C_2\dot\theta_1 + l_2(\dot\theta_1+\dot\theta_2) \\ 0 \end{bmatrix} $$

$$ = \begin{bmatrix} C_3 l_1 S_2\dot\theta_1 - S_3\left(l_1 C_2\dot\theta_1 + l_2(\dot\theta_1+\dot\theta_2)\right) \\ S_3 l_1 S_2\dot\theta_1 + C_3\left(l_1C_2\dot\theta_1 + l_2(\dot\theta_1+\dot\theta_2)\right) \\ 0 \end{bmatrix} $$

$$ {}^4\omega_4 = {}^3\omega_3 $$

$$ {}^4V_4 = \underbrace{{}^4_3R}_{I}\left({}^3V_3 + {}^3\omega_3 \times \overbrace{{}^3P_4}^{{}^3P_{TOE}}\right) = \begin{bmatrix} C_3 l_1 S_2\dot\theta_1 - S_3\left(l_1C_2\dot\theta_1 + l_2(\dot\theta_1+\dot\theta_2)\right) - {}^3P_{TOE_Y}(\dot\theta_1+\dot\theta_2+\dot\theta_3) \\ S_3 l_1 S_2\dot\theta_1 + C_3\left(l_1C_2\dot\theta_1 + l_2(\dot\theta_1+\dot\theta_2)\right) \\ 0 \end{bmatrix} $$

RECALL $L_1 = 500\text{cm}$ $L_2 = 400\text{cm}$ $P_{TOE} = 150\text{cm}$

USING TRIG IDENTITIES

$$ {}^4J(\theta) = \begin{bmatrix} 500\sin(\theta_2-\theta_3) - 400S_3 - 150 & -400S_3 - 150 & -150 \\ 500\cos(\theta_2-\theta_3) + 400C_3 & 400C_3 & 0 \\ 1 & 1 & 1 \end{bmatrix} $$

THE REASON FOR 1's IN BOTTOM ROW

$$ \begin{bmatrix} V_x \\ V_y \\ \omega \end{bmatrix} = {}^4J \begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot\theta_3 \end{bmatrix} \qquad \omega = \dot\theta_1 + \dot\theta_2 + \dot\theta_3 $$

WE WILL SOLVE

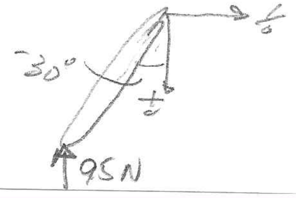
$$ \tau = {}^4J^T\,{}^4F \qquad {}^4F = {}^4_0R\,{}^0F \qquad {}^0_3R = {}^0_4R = \begin{bmatrix} C_{123} & -S_{123} & 0 \\ S_{123} & C_{123} & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

$$ {}^4_0R = \begin{bmatrix} C_{123} & S_{123} & 0 \\ -S_{123} & C_{123} & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

$$ {}^4F = \begin{bmatrix} .866 & -.5 & 0 \\ .5 & .866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -95 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -82.27 \\ -47.5 \\ 0 \end{bmatrix} $$

$$ \theta_{123} = 10.5 - 44 + 3.5 = -30^\circ $$

$$ {}^4_0R = \begin{bmatrix} C(-30) & S(-30) & 0 \\ -S(-30) & C(-30) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} .866 & -.5 & 0 \\ .5 & .866 & 0 \\ 0 & 0 & 1 \end{bmatrix} $$



5.22 CONTINUED

$$\tau = {}^4J^T \, {}^4F$$

PLUG IN θ's

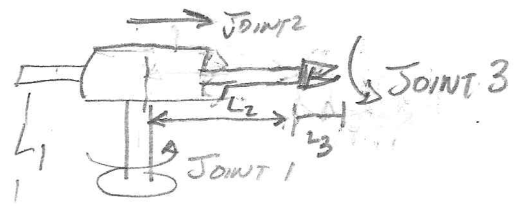
$$\tau = \begin{bmatrix} -543.94 & -174.74 & -150 \\ 736.42 & 399.73 & 0 \\ 1 & 1 & 1 \end{bmatrix}^T \begin{bmatrix} -82.27 \\ -47.5 \\ 0 \end{bmatrix}$$

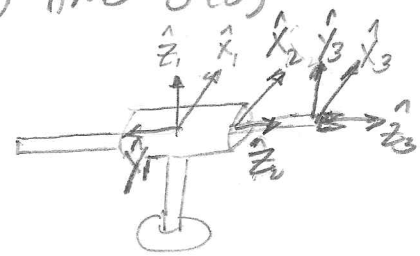
$$= \begin{bmatrix} -543.94 & 736.42 & 1 \\ -174.74 & 399.73 & 1 \\ -150 & 0 & 1 \end{bmatrix} \begin{bmatrix} -82.27 \\ -47.5 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -543.94(-82.27) + 736.42(-47.5) \\ -174.74(-82.27) + 399.73(-47.5) \\ -150(-82.27) \end{bmatrix}$$

$$= \begin{bmatrix} 9,769 \\ -4,611 \\ 12,340 \end{bmatrix} \text{ N·cm}$$

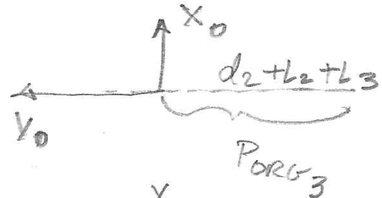
5.30 GIVE THE 3X3 JACOBIAN THAT CALCULATES THE LINEAR VELOCITY OF THE TOOL TIP FROM THE THREE JOINT RATES OF THE MANIPULATOR OF EXAMPLE 3.4. FIND ${}^0J(\theta)$ AND ${}^3J(\theta)$



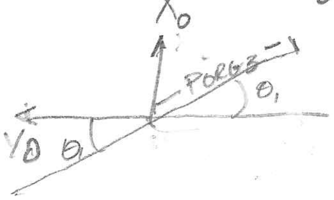


RECALL $${}^0J = \begin{bmatrix} \frac{\partial PORG_{3X}}{\partial \theta_1} & \frac{\partial PORG_{3X}}{\partial d_2} & \frac{\partial PORG_{3X}}{\theta_2} \\ \frac{\partial PORG_{3Y}}{\partial \theta_1} & \frac{\partial PORG_{3Y}}{\partial d_2} & \frac{\partial PORG_{3Y}}{\partial \theta_2} \\ \frac{\partial PORG_{3Z}}{\partial \theta_1} & \frac{\partial PORG_{3Z}}{\partial d_2} & \frac{\partial PORG_{3Z}}{\partial \theta_2} \end{bmatrix}$$

THIS IS DIFFICULT TO SEE
(LOOK DOWN ON THE MANIPULATOR)



ROTATION θ_2 DOES NOT AFFECT END OF $PORG_3$



$${}^0P_{ORG_3} = \begin{bmatrix} (d_2+L_2+L_3)S_1 \\ -(d_2+L_2+L_3)C_1 \\ 0 \end{bmatrix}$$

TAKE PARTIAL DERIVATIVES ABOVE

$${}^0J = \begin{bmatrix} (d_2+L_2+L_3)C_1 & S_1 & 0 \\ (d_2+L_2+L_3)S_1 & -C_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

5.30 CONTINUED

FOR THIS SYSTEM

$$
{}^0_3R = \begin{bmatrix} C_1 & -S_1 & 0 \\ S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 \\ S_3 & C_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} C_1C_3 & -C_1S_3 & S_1 \\ S_1C_3 & -S_1S_3 & -C_1 \\ S_3 & C_3 & 0 \end{bmatrix}
$$

$$
{}^3J = {}^0_3R^T\,{}^0J = \begin{bmatrix} C_1C_3 & S_1C_3 & S_3 \\ -C_1S_3 & -S_1S_3 & C_3 \\ S_1 & -C_1 & 0 \end{bmatrix} \begin{bmatrix} (d_2+L_2+L_3)C_1 & S_1 & 0 \\ (d_2+L_2+L_3)S_1 & -C_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

$$
= \begin{bmatrix} C_1C_3(d_2+L_2+L_3)C_1 + S_1C_3(d_2+L_2+L_3)S_1 & \overset{0}{\cancel{S_1C_1C_3}} - \overset{0}{\cancel{S_1C_1C_3}} & 0 \\ -C_1S_3(d_2+L_2+L_3)C_1 - S_1S_3(d_2+L_2+L_3)S_1 & \overset{0}{\cancel{-C_1S_3S_1}} + \overset{0}{\cancel{S_1S_3C_1}} & 0 \\ \underbrace{S_1(d_2+L_2+L_3)C_1 - C_1(d_2+L_2+L_3)S_1}_{0} & \underbrace{S_2+C_2}_{=1} & 0 \end{bmatrix}
$$

$$
= \begin{bmatrix} C_3(d_2+L_2+L_3)(C_1^2+S_1^2) & 0 & 0 \\ -S_3(d_2+L_2+L_3)(C_1^2+S_1^2) & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}
$$

$$
\underline{\underline{{}^3J = \begin{bmatrix} C_3(d_2+L_2+L_3) & 0 & 0 \\ -S_3(d_2+L_2+L_3) & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}}
$$